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A generalized Bell-Inequality and Decoherence for the $K^0\bar{K}^0$-System

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Abstract

First a generalized Bell-inequality for different times and for different quasi-spin states is developed. We focus on special quasi-spin eigenstates and times. The inequality based on a local realistic theory is violated by the CP-violating parameter [1], if the quantum theory is used to recalculate the probabilities. Next the quantum mechanical probabilities are modified by the decoherence approach which enables the initial state to factorize spontaneously. In this way we get a lower limit for the decoherence parameter ζ, which measures the degree of decoherence. This result is compared with the experimental value [2, 3] of the decoherence parameter ζ deduced from the data of the CPLEAR-experiment [4].

Key-Words: neutral kaons, Bell-inequality, decoherence parameter, CP-violation, locality

1 Introduction

Tests of quantum mechanics (QM) against local realistic hidden variable theories are of great interest since the first formulation of the EPR paradox by Einstein, Podolski and Rosen in 1935 [5]. J.S. Bell [6] proved in 1964 the important theorem that a whole class of local realistic hidden variable theories cannot reproduce all statistical predictions of QM.
Another approach was done by Wigner. He started from a pure set-theoretical point of view, where he - simply spoken - counted all super-pure states, respectively, all possible measurable results of such states.
However, to test such inequalities experimentally, a better approach is the Bell-CHSH-inequality (Clauser-Horne-Shimony-Holt) [8, 9, 10], because it doesn't require perfect anti-correlation nor perfect measurement apparatus. These experiments using correlated photons have been done by many groups [11, 12] and the results agree with the quantum mechanical predictions.
In recent years people started to consider massive EPR-like correlated particle systems, see for instance Ref.[13] to Ref.[22]. One of them is the neutral kaon system, where a to the photon case similar singlet-state can be produced.
In this work we are going to develop a Bell-CHSH-inequality based on a hidden variable theory for the neutral entangled kaon system where both the times and the quasi-spin eigenstates can be differed. This turns out to be a generalization, which allows us to handle existing Bell-inequalities derived with different methods in an uniform way.
The variation of the times when a special quasi-spin eigenstate is measured is analogous to the spin-case, however, the theoretical calculations show that a Bell-CHSH-inequality cannot be violated by quantum mechanics due to the specific constants of that neutral kaon system [16]. If the quantity $x = \frac{2\Delta m}{\gamma_S}$ was a factor 4.3 smaller than the experimental $x \approx 1$, the Bell-CHSH-inequality would be violated [3].
But if one allows to vary the quasi spin eigenstate the Bell-CHSH-inequality can be violated by quantum mechanics. Choosing three special eigenstates, namely $|K_S\rangle$, $|\bar{K}^0\rangle$ and $|K_1^0\rangle$, one can transform the Bell-CHSH-inequality into a Wigner-type inequality. Then by inserting the quantum mechanical joint probabilities into this Wigner-type inequality one gets an inequality on the CP-parameter ε [1]. This Wigner-type inequality is violated by the experimental value of the CP-parameter ε [26].
We want to bring to the readers attention the importance of using the correct time evolution for the derivation of the quantum mechanical probabilities, see section 3. Our Bell-CHSH-inequality differs from the inequality in literature, e.g., [18], we obtain an additional correlation function, since we deal with a unitary time evolution.
Next we consider another method, the decoherence method, which has been developed for the massive neutral correlated kaon-system in [2, 3, 19] and for the massive correlated neutral B-mesons [21]. This method modifies quantum mechanics in the way that the quantum mechanical interference term is multiplied by by a factor $(1 - \zeta)$, where

the decoherence parameter ζ equal 0 refers to QM and ζ equal 1 refers to a vanishing interference term, this case is called total decoherence or Furry's hypothesis[1] or spontaneous factorization. In the neutral kaon system the value of the decoherence parameter ζ has been calculated using the data of the CPLEAR-experiment [4] at CERN, the results are published in [2].
In order to find a closer connection between the decoherence approach and local deterministic theories we use quantum probabilities derived with that simple manipulation to recalculate the Wigner-type inequality.

2 The Bell-CHSH-inequality for the K-mesons

Here we will derive a Bell-CHSH-inequality based on locality, realism and induction for different times and for arbitrary quasi spin states in the massive EPR-correlated neutral kaon system. First we will focus on the similarities and differences of the photon system compared with the neutral kaon system.

2.1 Introductory considerations

Testing the predictions of quantum mechanics against those of any local deterministic hidden variable theory presents some analogies but also significant practical and conceptual differences with respect to the corresponding problem in the case of spin variables. The differences derive from two specific features.

1. First, while in the spin or photon case one can devise a test to check whether a spin $\frac{1}{2}$-particle is or is not in any chosen spin state $\alpha|\Uparrow_n\rangle + \beta|\Downarrow_n\rangle$, there is no analogous way to test whether the system is in the linear superposition $\alpha|K^0\rangle + \beta|\bar{K}^0\rangle$. However, as done in [1, 18] we will assume that the four following superpositions of the strangeness eigenstates can be measured by a gedanken experimentator

$$
\begin{aligned}
\text{Mass-eigenstates:} \qquad |K_S\rangle &= \frac{1}{N}\big\{p|K^0\rangle - q|\bar{K}^0\rangle\big\} \\
|K_L\rangle &= \frac{1}{N}\big\{p|K^0\rangle + q|\bar{K}^0\rangle\big\} \\
CP\text{-eigenstates:} \qquad |K_1^0\rangle &= \frac{1}{\sqrt{2}}\big\{|K^0\rangle - |\bar{K}^0\rangle\big\} \\
|K_2^0\rangle &= \frac{1}{\sqrt{2}}\big\{|K^0\rangle + |\bar{K}^0\rangle\big\} \qquad (1)
\end{aligned}
$$

[1]This hypothesis, actually, should be called Schrödinger's hypothesis, first because he published such a factorization of an entangled state one year [24] before W. Furry [23] and second he stated that such a happening really could occur.

with $p = 1 + \varepsilon$, $q = 1 - \varepsilon$, $N^2 = |p|^2 + |q|^2$ and ε being the complex CP-violating parameter. So far, the following derivations and conclusions belong to a gedanken experiment, but in the end we just need the information of the CP-violating parameter which can be measured by an arbitrary experiment not necessarily dealing with entangled particles.

2. The second important difference drives from the fact that while in the spin-case the direct product space $H^1_{spin} \otimes H^2_{spin}$ is sufficient to account for all spin properties of the entangled system, this is *not* true for the neutral kaon case.

   As a consequence the norm of the component on such a space decreases with time, if one doesn't take the decay products into account; that is the main difference between the works of, e. g., [18] and [16].

   Indeed, we want to emphasize that in the case under consideration the state vector acquires by time evolution components on the manifold of the decay products which are orthogonal to the product space $H^r_{kaon} \otimes H^l_{kaon}$. Then we get a unitary time evolution, which leads to additional terms in the resulting Wigner-type inequality, see section 4.

2.2 Requirements of locality

In the case of spin variables one can derive the Bell-CHSH-inequality [8, 9], for the averaged values of spin directions along arbitrary quantization directions n and m. The analogue of the free choice of the spin directions is, in the kaon case, the free choice of the times at which measurements aimed to detect the quasi spin states of the meson. But, in addition, we have the freedom of choosing the quasi spin state of the meson, the strangeness-eigenstate, the mass-eigenstate or the CP-eigenstate.
The locality assumption requires then that the results at one side be completely independent of the time and the choice of the quasi spin eigenstates at which the measurement at the other side is performed. To define the appropriate correlation functions to be used in Bell's inequality, one considers an observable $O^r(k_n, t_a)$ on the right side, which assumes the value $+1$ if the measurement at time point t_a gives the quasi spin eigenstate k_n and the value -1 if the quasi spin eigenstate k_n is not found. In terms of such an observable we can define the correlation function $O(k_n t_a; k_m t_b)$, which takes the value $+1$ both when a k_n at t_a and a k_m at t_b was detected or when no k_n and no k_m was detected. In the case that only one of the desired quasi spin eigenstate has been found, no matter at which side, the correlation function takes the value -1.
The locality assumption implies then that $O(k_n t_a, k_m t_b)$, in a specific individual experiment, equals the product of $O^r(k_n, t_a)$ and $O^l(k_m, t_b)$:

$$O(k_n t_a; k_m t_b) \;\; = \;\; O^r(k_n, t_a) \;\cdot\; O^l(k_m, t_b). \tag{2}$$

From this equation taking one derives immediately

$$|O(k_n t_a; k_m t_b) - O(k_n t_a; k_{m'} t_c)| \; + \; |O(k_{n'} t_d; k_{m'} t_c) + O(k_{n'} t_d; k_m t_b)| \; = 2 \tag{3}$$

with $k_n, k_m, k_{m'}$ and $k_{n'}$ being arbitrary quasi spin eigenstates of the meson and t_a, t_b, t_c and t_d four different times.
Let us now consider a sequence of N identical measurements, and let us denote by O_n the value taken by O in the i-th experiment. The average is then given by

$$M(k_n t_a; k_m t_b) \;\; = \;\; \frac{1}{N} \sum_{i=1}^{N} O_i(k_n t_a; k_m t_b) \tag{4}$$

and satisfies the Bell-CHSH-inequality [8, 9]

$$\begin{aligned} &|M(k_n t_a; k_m t_b) - M(k_n t_a; k_{m'} t_c)| + |M(k_{n'} t_d; k_{m'} t_c) + M(k_{n'} t_d; k_m t_b)| \; \leq \\ &\frac{1}{N} \sum_{i=1}^{N} \big\{ |O_i(k_n t_a; k_m t_b) - O_i(k_n t_a; k_{m'} t_c)| \; + \; |O_i(k_{n'} t_d; k_{m'} t_c) + O_i(k_{n'} t_d; k_m t_b)| \; = \; 2. \end{aligned} \tag{5}$$

3 How to derive the quantum probabilities?

We will follow here the formalism described in [16], but generalize it using both different times and arbitrary quasi spin-states. The complete evolution of the mass-eigenstates is described by a unitary operator $U(t,0)$ whose effect can be written as

$$U(t,0)\,|K_{S,L}\rangle \;\; = \;\; e^{-i\lambda_{S,L} t}\,|K_{S,L}\rangle + |\Omega_{S,L}(t)\rangle \tag{6}$$

where $|\Omega_{S,L}(t)\rangle$ describes the decay products. Thus we operate in a complete Hilbert space in opposite to, i.e., [18].
The initial state of the strong decay of the Φ-meson, $J^{PC} = 1^{--}$ into a pair of neutral K-mesons is given in the $K^0\bar{K}^0$-basis and $K_S K_L$-basis choice by

$$\begin{aligned} &K^0\bar{K}^0\text{-basis:} \;\; |\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \big\{ |K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle \big\} \\ &K_S K_L\text{-basis:} \;\; |\psi(t=0)\rangle = \frac{N^2}{2pq\sqrt{2}} \big\{ |K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle \big\}. \end{aligned} \tag{7}$$

The state at time t is then obtained from (7) by applying to it a unitary operator which is the direct product

$$U(t,0) \;=\; U_r(t,0)\cdot U_l(t,0) \tag{8}$$

of the operators $U_r(t,0)$ and $U_l(t,0)$ acting on the space of the right and the left mesons in accordance with (6).
According to standard quantum mechanics we will now evaluate the probabilities of finding different quasi spin eigenstates in measurements at two different times t_r and t_l; without loss of generality $t_r > t_l$. We denote by $P_r(k_n)$ the projection operator on the right side projecting the quasi spin-eigenstate k_n, so that, e. g. $P_r(K^0) = |K^0\rangle_r\langle K^0|_r$. As usual the projection operator $Q_r(k_n) = 1 - P_r(k_n)$ acts on the manifolds orthogonal to those associated to $P_r(k_n)$.
Starting from the initial state (7) one gets with (8) at time t_l the state

$$|\psi(t_l,t_l)\rangle \;=\; U(t_l,0)|\psi(t=0)\rangle \;=\; U_r(t_l,0)U_l(t_l,0)|\psi(t=0)\rangle. \tag{9}$$

If we now measure a k_m on the left side at t_l this state yields reduction to the state

$$|\tilde{\psi}(t_l,t_l)\rangle \;=\; P_l(k_m)|\psi(t_l,t_l)\rangle. \tag{10}$$

Now we need to evaluate this state to the time t_r and project on the right state

$$|\tilde{\tilde{\psi}}(t_r,t_l)\rangle \;=\; P_r(k_n)U_r(t_r,t_l)P_l(k_m)|\psi(t_l,t_l)\rangle. \tag{11}$$

The state of that equation (11) gives the probability of finding a mesons in the state k_n on the right side at time t_r and a k_m state at the left side at time t_l. Such a state, taking into account the unitarity and the composition laws of the operator U as well as the fact that operators referring to the different (right and left) Hilbert spaces commute, coincides with the state

$$|\psi(t_r,t_l)\rangle \;=\; P_r(k_n)P_l(k_m)U_r(t_r,0)U_l(t_l,0)|\psi(t=0)\rangle. \tag{12}$$

In this publication we will consider probabilities of finding or not finding a specific quasi spin state in a specific measurement. Derivations of the corresponding probabilities for such a process can be done by using formula (12) with the operators Q replacing the operators P, where required.
For example, the joint probabilities that in two measurements at t_r and t_l a quasi-spin state k_n is detected (Y) and a quasi-spin state k_m is not detected (N) is given by

$$P_{n,m}(Y t_r, N t_l) \;=\; ||P_r(k_n)Q_l(k_m)U_r(t_r,0)U_l(t_l,0)|\psi(t=0)\rangle||^2. \tag{13}$$

4 The general Bell-CHSH-inequality

The derivation of the quantum mechanical probabilities for finding on the right side at t_a a quasi-spin state k_n or not and on the left side at t_b a quasi-spin state k_m or not has been shown in the last section. Hence, we can write the quantum mechanical expectation value for finding an arbitrary state k_n at time t_a on the right side and a state k_m at time t_b on the left side

$$\begin{aligned} & M^{QM}(k_n t_a, k_m t_b) = \\ & P_{n,m}(Yt_a, Yt_b) + P_{n,m}(Nt_a, Nt_b) - P_{n,m}(Yt_a, Nt_b) - P_{n,m}(Nt_a, Yt_b) \end{aligned} \tag{14}$$

where $P_{n,m}(Yt_a, Yt_b)$ is the probability of finding a k_n at t_a on the right side and finding a k_m at t_b on the left side; $P_{n,m}(Yt_a, Nt_b)$ denotes the case when we find a k_n at t_a, but our detector doesn't detect a k_m at t_b.
Further we can use that the sum of the statistical frequencies of the results (Y, Y), (N, N), (Y, N) and (N, Y) is one for all times, so Eq.(14) can be rewritten to

$$M^{QM}(k_n t_a, k_m t_b) = -1 + 2\big\{P_{n,m}(Yt_a, Yt_b) + P_{n,m}(Nt_a, Nt_b)\big\}. \tag{15}$$

Setting this expression into the Bell-CHSH-inequality (5) we get the following inequality

$$\begin{aligned} |P_{n,m}(Yt_a, Yt_b) + P_{n,m}(Nt_a, Nt_b) - P_{n,m'}(Yt_a, Yt_c) - P_{n,m'}(Nt_a, Nt_c)| \;\leq \\ 1 \pm \big\{ -1 + 1 \,\{\quad P_{n',m}(Yt_d, Yt_b) + P_{n',m}(Nt_d, Nt_b) \\ + P_{n',m'}(Yt_d, Yt_c) + P_{n',m'}(Nt_d, Nt_c)\}\big\}. \end{aligned} \tag{16}$$

4.1 The Wigner-type inequality

To derive from this Bell-CHSH-inequality (16) the Wigner-type inequality we have to choose the upper sign + and we get

$$\begin{aligned} P_{n,m}(Yt_a, Yt_b) \quad \leq \quad & P_{n,m'}(Yt_a, Yt_c) + P_{n',m}(Yt_d, Yt_b) + P_{n',m'}(Yt_d, Yt_c) \\ & + h(n, m, n', m'; t_a, t_b, t_c, t_d) \end{aligned} \tag{17}$$

with

$$\begin{aligned} h(n, m, n', m'; t_a, t_b, t_c, t_d) \quad = \quad & -P_{n,m}(Nt_a, Nt_b) + P_{n,m'}(Nt_a, Nt_c) + P_{n',m}(Nt_d, Nt_b) \\ & + P_{n',m'}(Nt_d, Nt_c) \end{aligned} \tag{18}$$

Setting $t_a = t_b = t_c = t_d = t = 0$ the function $h(n, m, n', m'; t = 0)$ is equal to

$$\begin{aligned} h(n, m, n', m'; t = 0) &= -P_{n,m}(Y, Y)|_{t=0} + P_{n,m'}(Y, Y)|_{t=0} + P_{n',m}(Y, Y)|_{t=0} \\ &\quad + P_{n',m'}(Y, Y)|_{t=0} \end{aligned} \tag{19}$$

because $P(Y, Y)|_{t=0} \equiv P(N, N)|_{t=0}$. To get rid of the fourth probability we use the anti-correlation of the entangled system, setting n' equal to m' this probability becomes zero. Thus we derive the following Wigner-type inequality at $t = 0$

$$\boxed{P_{n,m}(Y, Y)|_{t=0} \;\leq\; P_{n,n'}(Y, Y)|_{t=0} + P_{n',m}(Y, Y)|_{t=0}}$$

This inequality was found by Uchiyama [1] by a set-theoretical approach. He showed that for choosing

$$\begin{aligned} |k_n\rangle &= |K_S\rangle \\ |k_m\rangle &= |\bar{K}^0\rangle \\ |k_{n'}\rangle &= |K_1\rangle \end{aligned} \tag{20}$$

the Wigner-type inequality is violated by the CP-violating parameter ε:

$$Re\{\varepsilon\} \;\leq\; |\varepsilon|^2. \tag{21}$$

Higher orders in the CP-violating parameter of ε are neglected. This Wigner-type inequality (21) is obviously violated by the experimental value of ε, having an absolute value of about 10^{-3} and a phase of about $45°$.
If we would replace the anti-kaon with a kaon, thus choose our three states in the following way

$$\begin{aligned} |k_n\rangle &= |K_S\rangle \\ |k_m\rangle &= |K^0\rangle \\ |k_{n'}\rangle &= |K_1\rangle \end{aligned} \tag{22}$$

we end with the inequality

$$-\,Re\{\varepsilon\} \;\leq\; |\varepsilon|^2. \tag{23}$$

which is obviously not violated.

On the other hand replacing the short living state $|K_S\rangle$ by the long living state $|K_L\rangle$ and the CP-eigenstate $|K_1\rangle$ by $K_2\rangle$ we find the same inequality as (21).
Note, when deriving the Wigner-type inequality (21) from a set theoretical approach the orthogonality of the mass-eigenstates is not needed; the states are only mutually orthogonal of the order of $O(|\varepsilon|)$ but not of the order $O(|\varepsilon|^2)$. In this gedanken experiment, just one mass-eigenstate is sufficient, one could reformulate the approach by assuming to the mass-eigenstate $|K_S\rangle$ an orthogonal one, analogously to [18].
However, our aim is not to measure the states $|K_S\rangle$ and $|K_1^0\rangle$. The Wigner-type inequality for the choice (20) is not physical in the sense of experimentally testable, but using the quantum mechanical formalism we get an inequality (21) for a physical quantity, the CP-parameter ε. This result will be connected to a modified theory in section 5.

4.2 The Wigner-type inequality for equal times

Now we will consider the Wigner-type inequality of the previous section for times greater zero, but equal-time measurements. We have to pay attention to the correction function $h(n, m, n', m'; t)$.
For the choice (20) the correction function $h(n, m, n', m'; t)$ is given by

$$\begin{aligned} h(K_S, \bar{K}^0, K_1^0, K_1^0; t) = 2 + \frac{1}{1-x^2}\Big\{ &\frac{-2}{1+|\varepsilon|^2} e^{-\gamma_S t} + \frac{-2|\varepsilon|^2}{1+|\varepsilon|^2} e^{-\gamma_L t} \\ &+ (1-x^2)\frac{|\varepsilon|^2 - Re\{\varepsilon\}}{1+|\varepsilon|^2} e^{-2\gamma t} + 2x^2 \cos(\Delta m t) \cdot e^{-\gamma t} \Big\} \end{aligned} \tag{24}$$

It turns out that the time-dependent Wigner-type inequality is only violated for times smaller than $t = 8 \cdot 10^{-4} \tau_S$, hence, for times larger this value the Wigner inequality is restored.

4.3 The Wigner-type inequality for different times

To avoid the fast increase of the correction function h the times can be chosen $t_a = t_c = t_d$ with $t_a \leq t_b$. The violation of the Wigner-type inequality is strongest for t_a close to zero; in this case a violation for t_b up to $4\tau_S$ can be found.

5 The Wigner-type inequality parameterized by the decoherence parameter ζ

Now we want to bring the decoherence parameter ζ into the game. We are going to recalculate the probabilities needed for the Wigner inequality (17), but with possible

decoherence, i.e., the interference term is multiplied by the factor $(1-\zeta)$ where ζ is the decoherence parameter, already used in [2, 3, 21, 19]. This simple modification gives the neutral kaon system the possibility of spontaneous factorization of the initial state (7).

The idea here is to determine which degree of decoherence, i.e., which value of the decoherence parameter ζ is sufficiently large, to restore the inequality (17). In this way we can relate the decoherence approach to a local realistic theory.

This procedure is basis dependent, thus it depends on which basis the spontaneous factorization occurs in, i.e. in which basis the cross products terms are affected by the decoherence. In the publications [2, 3] it has been demonstrated that using the experimental data of the CPLEAR experiment at CERN [4] the value of the decoherence parameter ζ can be calculated.

Further it has been demonstrated that there are two physical interesting basis choices, the $K_S K_L$- and the $K^0\bar{K}^0$-basis choice. We will start with the $K_S K_L$-basis choice and continue with the $K^0\bar{K}^0$-basis.

5.1 Calculations in the $K_S K_L$-basis

Denoting with k_n and k_m as usual the quasi-spin states the joint probability $P_{n,m}(Yt_r, Yt_l)$ can be derived according to QM - starting from the initial anti-symmetric state in the $K_S K_L$-basis (7) and using the formalism of section 3 - to

$$\begin{aligned}
P^{\zeta}_{n,m}(Yt_r, Yt_l) &= ||P_r(k_n)P_l(k_m)U_r(t_r)U_l(t_l)\frac{N^2}{2pq\sqrt{2}}\big\{|K_S\rangle_r|K_L\rangle_l - |K_L\rangle_r|K_S\rangle_l\big\}||^2 \\
= \;& \frac{N^4}{8|p|^2|q|^2}\cdot \\
& ||\langle k_n|U_r(t_r)|K_S\rangle\;\langle k_m|U_l(t_l)|K_L\rangle\;\;|k_n\rangle_r|k_m\rangle_l \\
& \qquad\qquad - \langle k_n|U_r(t_r)|K_L\rangle\;\langle k_m|U_l(t_l)|K_S\rangle\;\;|k_n\rangle_r|k_m\rangle_l||^2 \\
\equiv \;& \frac{N^4}{8|p|^2|q|^2}\cdot\Big\{ \\
& |\langle k_n|U_r(t_r)|K_S\rangle\;\langle k_m|U_l(t_l)|K_L\rangle|^2 + |\langle k_n|U_r(t_r)|K_L\rangle\;\langle k_m|U_l(t_l)|K_S\rangle|^2 \\
& -2\underbrace{(1-\zeta)}_{\text{Modification}}Re\{\langle k_n|U_r(t_r)|K_S\rangle^*\;\langle k_m|U_l(t_l)|K_L\rangle^*\;\langle k_n|U_r(t_r)|K_L\rangle\;\langle k_m|U_l(t_l)|K_S\rangle\}\Big\}
\end{aligned} \tag{25}$$

with $U_{r,l}(t_{r,l}) \equiv U_{r,l}(t_{r,l}, 0)$. However, if we calculate the joint probability $P^{\zeta}_{n,m}(Yt_r, Nt_l)$ it gets more complicating

$$
\begin{aligned}
P^{\zeta}_{n,m}(Yt_r, Nt_l) &= ||P_r(k_n)Q_l(k_m)U_r(t_r)U_l(t_l)\frac{N^2}{2pq\sqrt{2}}\{|K_S\rangle_r|K_L\rangle_l - |K_L\rangle_r|K_S\rangle_l\}||^2 \\
= \quad & \frac{N^4}{8|p|^2|q|^2}\cdot \\
& ||\Big(\langle k_n|U_r(t_r)|K_S\rangle \;\; |k_n\rangle_r U_l(t_l)|K_L\rangle_l - \langle k_n|U_r(t_r)|K_S\rangle \; \langle k_m|U_l(t_l)|K_L\rangle \;\; |k_n\rangle_r|k_m\rangle_l\Big) \\
& -\Big(\langle k_n|U_r(t_r)|K_L\rangle \;\; |k_n\rangle_r U_l(t_l)|K_S\rangle_l - \langle k_n|U_r(t_r)|K_L\rangle \; \langle k_m|U_l(t_l)|K_S\rangle \;\; |k_n\rangle_r|k_m\rangle_l\Big)||^2 \\
\equiv \quad & \frac{N^4}{8|p|^2|q|^2}\cdot\Big\{ \\
& |\Big(\dots\Big)|^2 + |\Big(\dots\Big)|^2 \\
& -2\underbrace{(1-\zeta)}_{\text{Modification}} Re\{\Big(\dots\Big)^*\Big(\dots\Big)\}\Big\}.
\end{aligned} \tag{26}
$$

Deriving the ζ-terms for all probabilities under consideration and setting $t = 0$ the correction function $h_{\zeta}^{K_S K_L}$ (18) which includes all (N, N) probabilities gives

$$
\begin{aligned}
& h_{\zeta}^{K_S K_L}(K_S, \bar{K}^0, K_1^0, K_1^0; t = 0) = \\
& \qquad \frac{-Re\{\varepsilon\} + |\varepsilon|^2}{2(1+|\varepsilon|^2)} + \frac{\zeta}{1-x^2}\left\{\frac{x(1+x)}{4} + \frac{|\varepsilon|^2}{(1+|\varepsilon|^2)^2}\right\}
\end{aligned} \tag{27}
$$

with $x = \frac{2Re\{\varepsilon\}}{1+|\varepsilon|^2}$. Note, in opposite to pure quantum mechanics for this modified quantum theory the probability $P^{\zeta}_{n,n}(Y,Y)|_{t=0}$ and $P^{\zeta}_{n,n}(N,N)|_{t=0}$ is not necessarily zero for $\zeta \neq 0$.

For further simplification we will approximate the CP-violating parameter ε by setting the phase equal 45° and the parameters[2] $\eta_{00} = \eta_{+-} = \varepsilon$:

$$
\begin{aligned}
Re\{\varepsilon\} \;&\approx\; Im\{\varepsilon\} \geq 0 \\
&\Longrightarrow |\varepsilon|^2 \approx 2Re^2\{\varepsilon\} := 2u^2.
\end{aligned} \tag{28}
$$

[2]This is the superweak-model, which assumes ε' to be 0 [25]. However ε' not equal zero wouldn't effect our results.

Thus the approximate numerical value of $|\varepsilon|$ is $(2.28 \pm 0.019) \cdot 10^{-3}$ with a phase of $45°$ [25].
The calculation of all with the decoherence parameter modified probabilities of inequality (17) gives the following inequality for the decoherence parameter ζ to the order $O(u^2)$

$$\frac{u - 2u^2}{u + 6u^2} = 0.987 \leq \zeta. \tag{29}$$

This is our important result, it is of great interest because to satisfy this inequality the decoherence-parameter ζ has to be very close to 1; or in other words the interference term has to vanish. Hence, Furry's hypothesis or spontaneous factorization has to take place totally. This means in our case that after the creation of the entangled pair the initial state vector (7) factorizes in 50% of the cases in a short living state at the right side and in a long living state at the left side or in the other 50% of the cases vice versa.
Intuitively, we would have expected that there exist local realistic theories which allow at least partially an interference term, see for instance [15, 22]. Our result demands for a vanishing interference term, hence, the locality assumptions underlying this inequality forces the $K_S K_L$-interference term to vanish.
On the other hand this result can be compared with the results published in [2, 3]. In these publications a CPLEAR-experiment at CERN [4] in 1998 was considered. They produced entangled kaons and measured the strangeness-states. The average result for the decoherence parameter is $\zeta^{K_S K_L} = 0.13\{^{0.16}_{0.15}$. Within one standard deviation quantum mechanics $\zeta = 0$ is included, but total decoherence $\zeta = 1$ is excluded by many standard deviations.
This means for experimental reasons that $\zeta = 1$ is excluded and due to the Bell-inequality (29) a local realistic variable theory is impossible.

5.2 Calculations in the $K^0 \bar{K}^0$-basis

As we have seen in [2] the $K^0 \bar{K}^0$-basis choice is not a good one to discriminate between the quantum theory and Furry's hypothesis or spontaneous factorization. Let's see what we get if we recalculate the Wigner-type inequality ($t = 0$) this time starting by the $K^0 \bar{K}^0$-basis choice.
After a cumbersome calculation one gets for $t = 0$

$$u(1 - 2u^2) = 0.0016 \ \leq \ \zeta. \tag{30}$$

Hence, if ζ is just a little bigger than the small value 10^{-3}, the Wigner-type inequality will be restored and a local realistic theory which obeys the assumptions which lead to that Wigner-type inequality will be possible.

Again this result can be compared with the decoherence parameter for the $K^0\bar{K}^0$-basis choice derived with the help of the CPLEAR-data, which is $\zeta^{K^0\bar{K}^0} \sim 0.4 \pm 0.7$. In this basis choice the initial state (7) can factorize in a kaon at the right side and an anti-kaon at the left side or vice versa. Hence, within one standard deviation the quantum mechanical result $\zeta = 0$ is included, but also total decoherence $\zeta = 1$ is within one standard deviation.
On the other hand it should be possible to construct in that basis choice a local realistic theory which obeys the locality assumption of the derived inequality with a $K^0\bar{K}^0$-interference term which differs just by the order of $O(u)$ from the quantum mechanical result.

6 Conclusion

We develop a Bell-CHSH-inequality and simplify it to a Wigner-type inequality. Inserting the quantum mechanical probabilities into that inequality we find for $t = 0$ a Wigner-type inequality, the inequality of F. Uchiyama [1], which is violated by the value of the CP-violating parameter; thus we have a contradiction between a local hidden variable theory and the prediction of the quantum theory.
Next we consider a modified quantum theory, which describes possible spontaneous factorization of the initial state, where the measure of decoherence ζ has two limits, the quantum theory $\zeta = 0$ on one hand and a local theory $\zeta = 1$ on the other hand. Inserting such a modified quantum theory into a Bell-CHSH-inequality results in an inequality for the decoherence parameter ζ.
The simple modification was calculated for two basis choices, the K_SK_L-basis choice and the $K^0\bar{K}^0$-basis choice. The result for the first choice was that the decoherence parameter ζ had to be very close to 1 to fulfill the Wigner-type inequality and thus obey the locality assumption. Intuitively, we would have expected that there exist local realistic theories which allow at least partially an interference term in that basis choice, see for instance [15, 22].
However, comparing with the range of the decoherence parameter [2] derived with the data of the CPLEAR-experiment [4] we can exclude $\zeta = 1$.
In the $K^0\bar{K}^0$-basis choice the situation is different. The derived Wigner-type inequality (30) is satisfied for all ζs being larger than the small value of about 10^{-3}. Comparing that result with the range of the decoherence parameter ζ derived in [2], we learn that this basis choice doesn't give us - in contrast to the 'best' basis choice, K_SK_L - a powerful tool to distinguish between a realistic local theory obeying the Wigner-type inequality and QM on the other side.
Concluding, we have shown that there exists a best basis choice where one can distinguish clearly first to which extent decoherence could take place and second if within that range a local theory is possible. Thus with connecting the two different

approaches we can exclude a local realistic theory.

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